(i) Non-terminating Random Processes

By 'non-terminating random processes' we mean those phenomena whose development is not confined to a finite interval in time, but undergo fluctuation , change, and development for an indefinite period of time.

Such processes, which include the fluctuation of meterological factors (rainfall, flodd discharge, temperature, wind speed, pollution levels, etc) over long periods of time, involve us in a great many apparently insoluble problems when we try to subject them to a probabilistic analysis. Although we shall point out and attempt to clarify these problems, many of them must remain unsolved for the present.

The study of extremes most typifies the nature of the difficulties involved in studying non-terminating random processes, and it is the probabilistic prediction of extremes which is generally the most important aspect of their study. The other important aspect is the detection of past, and the prediction of future, qualitative changes in the nature of the process, or significant quantitative changes. This is in addition, of course, to the concrete physical, or experimental, study of the process, which is indispensable to a probabilistic analysis.

For such problems the theory of second moments is quite unsuitable. Firstly, this is because we shall be concerned with values which are widely divergent from the expected values. The 'tail' of a distribution can be defined in terms of the mean value only if the whole distribution holds to a constant analytical form, which is in general not the case. In short, to study extremes, extremes must be measured.

It is interesting to note that means of wind speed, taken over periods of a day, or shorter, and collected together in distributions covering a period of a year, or longer, tend to form distributions whose variate is best expressed as a ratio of the mean. On the other hand, when the variate is chosen as a mean over a period of longer duration, or is collected in batches covering a period of shorter duration, the distribution is one of the difference between the variate and the mean value. The implication of this is that, in general, even if the distribution of wind speed is considered to have a scale which is constantly varying, still the form, or shape, of the distribution must be considered to be constantly varying.

Further, with the long periods of time involved, the processes cannot sensibly be described in terms of the expected cross-products of the variate. For instance, if we are given that an extreme gust occurs at a certain time on a certain date, then the expected product of its value would for several hours perhaps be such as to indicate the increased likelihood of a following gust being extreme. However, since the extreme gust may, or may not, occur during a period of correspondingly high average speed, and periods of calm may intervene between related extreme gusts, the correlation coefficient would rapidly approach zero, and it would be difficult to express, by means of an expected cross-product, any information the known extreme would give us regarding the modified probability of a further extreme.

It has already been shown in the previous sections that the use of a second order moment approach somewhat restricts the freedom with which we can choose even analytical forms for the probability distribution. Such a restriction is inadmissible where we are concerned with extreme values. Further, second moment theory depends on the idea that there is a constant, or slowly varying mean value about which the variate is fluctuating. For most meterological phenomena this is not an accurate representation of the non-terminating process. The momentary values are determined by physical conditions which may, for instance, be reflected in the mean or expected value over an intermediate period. However, this, in turn, fluctuates according to other laws, and over still longer periods may not only fluctuate, but undergo permanent change.

Thus, it would seem that even a description of the process in terms of probability density functions will involve anomalies, since it will be difficult to find any probability distribution which remains constant throughout the development of the process. Still more difficult will it be to find a particular analytical form for that distribution.

Nevertheless, it shall always be neceessary to extrapolate beyond an essentially limited field of observation, and for this purpose we must choose some probability density function as a basis for extrapolation. Our aim shall be to increase as far as is possible the grounds for, and the accuracy of, extrapolation, and to reduce as far as is possible the necessary extent of extrapolation.

Before developing the basis for our study of extremes of wind speed, the following points regarding the method of study should be made.

(a) Probability, Frequency and Credibility

When we talk of the probability of an event, and the event is well-defined, it may be reasonably clear what is meant, but there exist a variety of definitions. Further, several quite different concepts may, or may not, be involved, and we should make the following distinctions.

Frequency of occurrence of an event.

Frequency gains its most precise definition in the Law of Large Numbers, but can be broadly defined as the proportion of the total number of future observations to which the actual number of occurrences of the event will converge, in probability. That is, with an indefinite increase in the number of observations the actual proportion will lie within some interval about this frequency, with a probability increasing with the number of observations.

Thus, the concept of frequency is dependent in the first place on the possibility of indefinitely increasing the number of observations. However, in some applications such an indefinite increase may be meaningless. For instance, it may be essentially the case that the conditions under which a variate is observed cannot endure forever, and may be constantly varying. When the frequency of peak values of variate is dependent on the value of an underlying mean, which is variable, then, unless the event can be re-expressed as relative to the conditions which determine it, we cannot define a simple frequency for the event.

In addition, if we are concerned with extremes, the number of observations is essentially small, no matter what the total size of the sample. Many of these difficulties are overcome by introducing the concept of a process.

A process.

If an observation is made at random (for instance if a point is chosen at random on the record of a given station, without regard to the year, season, or time of day), and if we can define the probability that a certain event is observed, then this probability may be deemed to be a frequency. If however, we are to make many observations, then the compound events which have been thus observed, with the inter-relations between the outcomes of different observations, may allow us to derive, or test, a process.

The process is an extension of the idea of frequency in which many frequencies, referring to similar or related events, are jointly, explicitly or implicitly, defined by the parameters of the process. The most significant aspect of a process is the manner in which it can describe multiple occurrences of the same outcome to a series of observations. However, the ability of a process to describe the occurrences of different events, such as the exceedences of different levels of a variate, is also important.

For instance, the Poisson process defines not only the frequency of an event, which is its parameter, but also the frequency of any interval between similar events. The Poisson process is of course quite distinct from a process in which the intervals between occurrences is normally distributed about a mean period, and these are both distinct from a process in which the moments at which the events occur are distributed about a series of equally spaced instants. Likewise, the concept of Return Periods used in the statistics of extremes is quite distinct from the concept of natural cycles.

The choice of a particular statistical process implies the acceptance of a particular physical model for the natural process generating the events under consideration.

The Poisson distribution of events rests on the fact that each event is the outcome an independent development, which is of a large number of similar processes.

The process in which the interval between two consecutive events is distributed about its mean value, rests on the existence of a single process of development, initiated at the moment of a previous occurrence, and operating continuously throughout the interval.

The process in which the instants at which the events occur, rather than the intervals, are distributed about a mean value, depends on the existence of a regular process defining the series of mean values whose effects are then modified by another process which is deemed to be random.

The normal distribution is derived from the independent activity of many processes which contribute additively to the value of the variable, each additive component having its unique distribution. If the independent processes combine their outcomes multiplicatively, then the resulting product has a log-normal distribution.

A variable is exponentially distributed if its value is equal or proportional to the number of independent conditions, having equal probability, which must all be satisfied in order that the value should be realised. For instance, the interval between the arrival of two telephone calls at an exchange may be seen to be exponentially distributed since, in order that its value should be t seconds or greater, there must be no call in each of the t seconds following the previous call. Each of the 'no call' events may be taken to have an equal probability, which is one minus the average frequency of arrivals.

The choice of a particular model, process, or probability function cannot be truly rationalised until it has both been verified in practice by measuring the empirical frequency distribution, and rationalised theoretically by establishing the physical model on which the statistical process is based.

Due to an imperfect understanding of meterology the author has been unable to derive such a physical model for the fundamental distribution of speeds which is suggested by the observed frequency distributions. This distribution will be described later. A study along these lines, to develop a physical model for the distribution would be enlightening.

If a set of observations are assumed to be derived from the action of a certain statistical process, then the relevant frequencies can be measured to derive estimates of the parameters of the process. Together with these estimates of the parameters, and the assumption that the process itself is correctly chosen, we may also estimate the probability distribution for the parameters of the process. Thus, it is not only possible to calculate the probability of an event in terms of the frequencies, but to each such calculation we must apply the probability attached to the frequencies themselves.

For instance, if we are determining the return period of a certain value of wind speed, and we may express our conclusion as three different return periods, all equally likely, given the data in hand, then, if we calculate the probability that the extreme will appear in the next, say, 20 years, the average of these three probabilities would be the best estimate we could make of the actual probability appropriate to the available data. Note that this technique would have the effect of weighting the result towards the conclusion based on the lowest return period alone.

However, since different processes imply different forms for the collection of all observed frequencies, such as the distribution of the intervals between events, it is possible also to question the applicability of the process itself, and this probability is over and above that assigned to the choice of parameters, since this latter probability was computed on the assumption that the correct process had been chosen. Thus, the problem of credibility will arise.

Credibility

Credibility, or confidence measure, or degree of confirmation, cannot usually be expressed in terms of frequencies at all.

Let us suppose that we wish to assign a probability to the truth of a staement which is meaningful and relates to precise points in time and space. Firstly, we cannot interpret this probability as indicating that the statement in such a degree true and in such a degree false. While one may consider such a conclusion, this has nothing to do with probability. Further, we cannot consider the probability to be the proportion of similar cases in which the statement is true, since we have prescribed that the statement refers to particular points in time and space.

For instance, if it were said that the credibility of the statement that wind records showing a certain configuration on Gumbel double-log chart did indeed derive from a process which will consistently obey the double-exponential distribution, was, say 20pc, and the same credibility was then assigned for the records taken from every station in the country, then it would seem optimistic to suppose that about 20pc of the stations did have a double exponential distribution. On the other hand if the credibility was computed as 20 pc on the basis of one record only, and all other stations showed very high credibilities, then, in the absence of geographical information which indicated that the one station in question was exceptional, we would be justified in accepting the hypothesis for that one station.

Thus, where credibility is concerned, no two similar cases can also be considered as independent cases. The credibility applies to the process as a whole, and where different observations have been made of a phenomenon at different places we must have solid grounds for asserting that the process applies at one location, but not at another.

So, a factor of credibility, referred to the widest possible field of data must be applied to any results obtained during an analysis. If the credibility for a process is significantly less than one, we must search for a new process.

Baye's formula for a hypothesis and an observation

It is interesting to note the following implication of Baye's formula. Let us suppose we have an hypothesis, H, being one of a set of possible hypotheses, H_i, and an observation O, likewise one of a set of possible observations, O_i. Now, it is usual to test the hypothesis by calculating the probability that would be assigned to the observation given that the hypothesis is true.

Baye's formula states,

$$P(O/H).P(H) = P(H/O).P(O) = P(H\&O)$$

Any one of the first four probabilities appearing in this formula may be computed if we are given the other three. For instance, we could take P(O) to be redundant, and compute it as follows,

$$P(O) = \sum_i P(O/H_i)P(H_i)$$

On the other hand, if we have a number of instances in which the same phenomenon is observed, we might be able to assign a value to P(O) directly.

Now, the form in which Baye's formula is usually applied, and which gives us the probability that we have selected the correct process, calculated on the basis of the observations, is

$$P(H/O) = P(H).P(O/H)/P(O)$$

We see that in order to compute P(H/O) we require in addition to P(O/H), P(H). That is, the observations may be considered to have modified the probability of the hypothesis, but we nevertheless require an initial value for P(H). The ratio by which the probability of the hypothesis is modified in the above equation could be called the degree of confirmation. If the degree of confirmation is greater than one, then the observations confirm the hypothesis, if it is less than one the observations give cause to doubt the hypothesis.

It is usual to use a suitably summarised form of the observations for O, such as a single parameter, the average value or dispersion, or curvature of a fitted line etc. This facilitates the operation, but, because of the ratio form used, it is not essential. Note that the set of hypotheses, H_i, if it is used, must be a set of mutually exclusive hypotheses, or in general, it must be so chosen as to be consistent with the methods used to calculate all the probabilities which appear in the formula.

It is difficult to see how we can assign a value to P(H) without first taking account of the observations, but this does correspond to the fact that a single case in which the hypothesis corresponds to the observations cannot confirm the hypothesis unless we begin with some reason to expect the hypothesis to be confirmed. For instance, we may know of observations relating to other instances of the same phenomenon, or there may be a physical argument supporting the hypothesis.

Consider, for example, the practice of quality control in industry, which often uses the Chi-squared distribution to test the mean of a supposed normal population. Typically, values of one or two per cent must be computed for P(O/H) in order to reject the hypothesis, H. This corresponds either to a great unwillingness to reject the hypothesis, or to a high value having been assigned to P(H) - the probability assigned to the hypothesis before the test is carried out.

This would bejustified if the test was one of many carried out on the same sample. In this case the results could be re-expressed to include all the observations as part of a single sample, and we return to the same point. Thus, it is clear that the decision to reject or accept the hypothesis will be as much a political decision as a mathematical and experimental operation.

The same question would arise if we were to construct an hypothesis to explain, for instance, a plotted set of observations which showed a very irregular and scattered form, by moulding the hypothesis precisely to the observations, ignoring the necessity for a physical argument. P(O/H) would then be very high, and a suitably chosen P(O) would be very low. However, such a low value would necessarily be assigned to P(H) that we would confirm with the use of Baye's formula the fact that P(H/O) was indeed small.

Recurrence and Occurrence

The words 'recurrence' and 'occurence' may be used to help clarify how the definition of an event determines the probability that should be assigned to the event, and thus influences the credibility we would assign to a hypothesis which is to be tested on the basis of observations of the event.

An event may have a very low frequency of recurrence, while in fact being a very common occurence. For instance, the event in which a set of plotted observations fit closely to a certain curve, or even to one of a well defined set of curves, while in fact the observations are not derived from the corresponding process, may be rare. On the other hand the event in which the constructor of the curve achieved a good fit may be a common occurence. The probability that the relevant configuration of plotted points will recur, while the hypothesis being tested is false, will be small if the constructor of the curve aimed only to fit the hypothesis to the observations. If this is the case, each fit he achieves in this way is a separate occurence of a common event, and the event cannot be said to have recurred.

That is, it would appear that if a hypothesis is constructed to fit a series of observations, then these observations cannot serve as a confirmation of the hypothesis. This applies to the process-type, and is critical where we are searching for a new process, but it does not apply to the parameters of the process, since an 'effective method' will exist for the determination of the parameters, and the computation of their significance, so long as the process is assumed valid. Thus, the distinction between a process and its parameters is vital. Note that the more parameters, or 'degrees of freedom' a process has, the less the degree of confirmation offered by an equally good fit.

On the other hand, it is essential that a process must be selected to fit the observations. It is contrary to any scientific method to ignore the observations while constructing the hypothesis simply in order that the observations may provide a means of confirming the hypothesis.

Assuming that the hypothesis will always be refined in the light of any observations which do not fit the hypothesis, the only basis we have for deciding whether the estimated degree of confirmation reflects a frequency of occurrence or a frequency of recurrence is the physical model underlying the statistical model. Similarly, it is only within the context of a physical understanding of a phenomenon that we may consider extrapolation beyond the field of observation, although the extent of the necessary recourse to physics varies with the nature and extent of the extrapolation.

(b) Extrapolation

From the beginning of an analysis a scientific worker must identify and define his field of observation, while striving to enlarge this field.

The field of observation will be bounded in time (records apply to a finite period which is not necessarily typical of previous or subsequent periods). It will be bounded in space - records will apply to certain geographic areas, not necessarily typical of all or any others. Each geographic location provides one observation of a process whose parameters apply to a single location. Observations will be bounded in the magnitude of the variate, and values outside the range of measured values could possibly be generated as a result of different physical processes from those which have been observed. The nature of the variate may also be characteristic of the set of observations; various aspects of the conditions under which the variate was measured may be untypical, or unsuitable for some applications. Such a case arises where an anemograph is placed at an unusual altitude which may make it difficult to compare the results with those taken at other stations. Also, we must be concerned with all correlations between these bounds, such as arises when we find that extreme values have been observed at some, but not all geographic locations, and we wish to discuss the occurrence of extreme values at a location where they have not yet been recorded.

The last class of extrapolation, in which it is only required to extend the assumption of the validity of the process to cover the coincidence of factors, under each of which, separately, it has been tested, which we shall call 'relative extrapolation', is more easily justified than its opposite ' absolute extrapolation'. Note that whether an extrapolation is deemed to be relative or absolute depends not so much on the extrapolation itself, but rather on the approach taken in carrying out the extrapolation.

Any statistical analysis should include data which is specifically designed to validate or invalidate extrapolation.

For instance, if two stations show a similar pattern over intermediate ranges of magnitude of the variate, then information regarding extremes obtained from one station could rationally be applied to the other if such information were lacking at that station.

Alternatively, if a random variable, C, is the sum of the random variables A and B, then a study of the observations of both A and B will allow us to apply relative extrapolation to a description of C. But on the other hand, if we take B to be the difference of C minus A, then this relative extrapolation becomes an absolute extrapolation.

Thus, although relative extrapolation is generally more easily rationalised, it must be based on a physical argument which will establish the continuity of the process throughout the field defined by the outer bounds of the observations, and by a statistical study of the interdependence of the components of the relative extrapolation.

Note that rational extrapolation is dependent on a high credibility for the process itself, and for the parameters of the process. For instance, if a curve is fitted through a set of plotted points, little error will arise from a poor fit or from excessive scatter so long as the curve is used only for interpolation on one of its axes. However, when the curve is extended, then any error in the fitting of the curve will be magnified. Further, the concept of a smooth curve towards which the plotted points are supposed to converge becomes somewhat questionable when this curve is extended to a point where there are no longer any plotted points.

(c) Seeking a new process.

If a set of observations do not fit a process, in probability, then we should seek a new process.

Although it may not always be the case that the original process is incorrect, or that a better process is suggested by the existing data, this remains the optimal procedure.

There can be no 'effective method' for the finding of a new process, but there are several procedures worth elaborating.

(A) Modification of the original process by increasing the number of parameters or by replacing the original variate by a function of the variate. For instance, a linear relation could be replaced by a parabolic relation.

(B) Selection of a new variate. It is usually impossible, for practical reasons, to choose a new field of observation, and measure a new variate, since the existence of a large body of reliable records is a prerequisite to any statistical study. The gathering of these records in past times would have been tailored to the requirements of the method of analysis that existed at that time, and this may be the only information available. However, it may be possible to form a new variate by combining two or more distinct observations.

(C) Repitition of the analysis after sorting the data by distinguishing between different classes of observation, and using different processes or different process parameters for each class. For instance, in the study of wind we could distinguish between summer, winter and equinox storms, between cyclones and other storms, or between large and small values. It is then necessary to weight the results according to the frequency and significance of each class.

For instance, if a cyclone had only once been recorded at a certain station, but this one produced the highest recorded speed, then we would be justified in removing that part of the record temporarily to study the remaining part of the record, and then making a separate study of the cyclone, drawing upon various relevant information that could be obtained from other stations. We would then have to assign a value to the frequency of such cyclones.

(D) Modification of the process to include space- and time- variant parameters. For instance, if some steady movement could be detected over the period of record, or some sudden change due to a change in the environment, then we could reduce each portion of the record accordingly, and re-adjust the results for extrapolation.

Any of the following findings may cause the investigator to adopt one of the above procedures.

(E) The observations show, in a number of instances, a systematic deviation from the form corresponding to optimal choice of the parameters. Except where one of the other procedures is recommended by a particular line of reasoning, (A) is worth trying. Such a systematic deviation can never be ignored however.

(F) Several classes of systematic deviation are observed in different instances of the process. If some physical basis may be found for the division into different classes, then (C) should be tried. Otherwise the deviations may be deemed to be 'unsystematic'.

(G) The observations show unsystematic variation, or 'scatter' from the assumed process, and this is in excess of the degree of scatter which follows from the process itself, or to severe to allow sufficient confirmation of the parameters of the process for extrapolation. (A) or (C) cannot reduce the scatter without a severe reduction in the 'confirmability' of the process. (B) should be tried.

(H) The observations show a systematic pattern, not necessarily contradicting the assumed statistical process, or reflected in the computed credibility, but not included in the process. For instance, some sort of correlation may be observed between the variables considered, or with other meaningful variables. The process should be expended or modified to include this, in effect, new data.

(I) The hypothesis fits the observations but does not lead to the solution of a particular problem of interest to the investigator. Preferably, the process should be expanded, or modified, retaining where possible the valuable components of the original hypothesis.

In conclusion, it should be noted that since the results obtained below relate to only one station there is no point in calculating credibility factors, since these depend chiefly on the consistency of the observed tendencies at different stations. It has been possible merely to indicate directions which show a greater or lesser potential for development.

Also, there are many instances where we have found directions which show a good possibility of development and refinement, but we have not carried these through, since the time involved would not be justified by the level which has been reached in other parts of the analysis.